ISSN 2231-0096

Platinum

A Peer Reviewed
National Multidisciplinary Journal
Volume - 14 Number - 2 April-June 2023

Editorial Office
Atharva Publications
Plot No.17, Devidas Colony
Varkhedi Road, Dhule - 424 001
www.atharvapublications.com
E-Mail : atharvapublications@gmail.com

Branch :
Circulation & Advertisement
Atharva Publications
Shop No. 2, Nakshatra Apartment, Shahu nagar Housing Society, Opp. Teli Samaj Mangal Karyalay, Jalgaon - 425001

Subscription Rates
Single Copy for reader Rs. 1000.00 or
US $ 100.00 Only (extra postage charge)
For Processing Charges of research Paper
Individuals Rs. 1500.00 (each research paper) Or
US $ 150.00
Institutions Rs. 3000.00 per annum Or Us $ 300.00

1. Editing of the research journal is processed without any remittance. The Selection and publication is done after recommendation of subject expert Refree.
2. Thoughts, Language vision and example in published research paper are entirely of author of research paper. It is necessary that both editor and editorial board are satisfied by the research paper. The responsibility of the matter of research paper is entirely of author.
3. Along with research paper it is compulsory to sent Memership form and copyright form.
4. In any condition if any National/ International university denies to accept the research paper published in the journal then it is not the responsiability of Editor, Managing Editor Publisher and Management.
5. Before re-use of published research paper in any manner, it is compulsory to take written acceptance form Managing Editor unless it will be assumed as disobedience of copyright rules.
6. All the legal undertaking related to this research journal are subjected to be hearable at Dhule Jurisdiction only.
7. The research journal will be sent by normal post. If the journal is not received by the author of research paper then it will not the responsibility of Editor and publisher. The amount or registered post should be given by the author of research paper. It will be not possible to sent second copy of research Journal.
8. Authors are requested to follow the author's Guide lines Contect Managing Editor - 9764694797

- For book reviews, please send two copies of the book (one for the Reviewer and other for the library of the journal) to the Managing editor.
- Donations of books /journals / cash / gift are welcome and will be gratefully acknowledged. All disputes concerning the journal will be settled in the court of Jalgaon, Maharashtra.

मेसर्स अथर्व पब्लिकेशन्स्च्यावतीने कार्यकारी संपादक श्री.युवराज माळी यांनी प्लॉट नं.१७, देविदास कॉलनी, धुळे-४२४ ००१ (महाराष्ट्र) येथे प्रकाशित केले व अविष्कार ग्राफीक्स , जळगाव येथे मुद्रित केले. मोबाईल : ९४०५२०६२३०. जळगाव (ऑ.) : ०२५७-२२३९६६६.

Index

Numerical Solution of Steady - State Heat Conduction Problem Using Grid Methods

S. R. Gaikwad
Department of Mathematics, J. D. M. V. P. Co. Op. Samaj's Arts, Commerce and Science College, Jalgaon, Jalgaon,
A. R. Gotmare
Department of Mathematics, Arts, Commerce and Science College, Jamner, Jalgaon,

Abstract

In this paper, numerical grid techniques have been used to solve steady state heat conduction problem with dirichlet boundary conditions in a rectangular domain. Finite element solution with triangular grid and finite difference solution with square grid are implemented here. Finally comparisons are made between the numerical solution obtained from the finite element and finite difference method solution.

Keywords : Finite element, Finite difference, Heat conduction, Numerical solution.

Finite Difference Method, Finite Volume Method, Finite Element Method, Boundary Element Method are many of mathematical techniques have been developed by mathematician and scientists for Numerical solution of engineering problems. Finite element analysis has now becomes integral part of computer aided engineering and is extensively used in analysis and design of many complex and real life system. The finite element method (FEM) can be viewed simply as a method of finding approximate solution of partial differential equation or a tool to transform partial differential equation into algebraic equation which are easily solved the continuous variation of the function concerned is represented by a set of values at points o a grid of intersecting lines in the finite difference method. The problem to which the method applied are specified by a partial differential equation, a solution region and boundary conditions. Also Finite Volume Method (FVM), Finite Element Method (FEM), Boundary Element Method (BEM) are numerical methods use in grid techniques. Out of these available numerical methods, finite element method and finite difference method are most useful numerical grid techniques. When the numerical grid methods are used in mathematical modelling first input is required for solving the problem. Also required universal constant, boundary conditions, constants, coefficients for particular problem. There are various options such as rectangular grid, triangular grid in two and three dimensional steady or unsteady state problem. The next part is generation of grid is important depending on the numerical method. The Grid can be generated manually or computer programming. Finally results obtained are processed in terms of tables, charts, graphs etc. Most of the development of these techniques will follow those found in electromagnetic book [5]. A solution of the heat equation and electromagnetics problem can be found in [6]. In the paper [3], the researcher discussed the spreadsheet implementation and advantages of spreadsheet. The spreadsheet are used for solving electrostatic boundary-value problem and spreadsheet can offer a reasonable tradeoff between user-defined programming and specific purpose software is presented in [2]. Advanced engineering mathematics [4] popular numerical grid techniques are FDM, FVM, FEM and BEM.

The finite element method (FEM) is a numerical grid technique for solving PDE. FEM is used widely for solving engineering problems in solid mechanics, aerospace, fluid mechanics, heat transfer, structural mechanics, automobiles and biomechanics. FEM and FDM are the most flexible and versatile grid methods for solving engineering problems.

In this paper section 2 gives the idea about the problem formulation. In section 3, the spreadsheet implementation and the essential basic steps have discussed. We have discussed the results in section 4. Finally, section 5 concludes a paper.

Problem Formulation

Consider steady state heat conduction in a rectangular region $0 \leq x \leq 4, 0 \leq y \leq 3$ subjected to the boundary conditions shown in the figure 1.

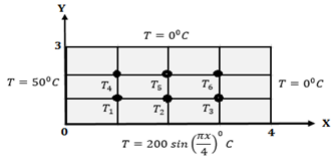

Figure 1: The Geometry and boundary conditions

[1]. Grid Numerical Methods

a. Finite Difference Method

The spreadsheet implementation of the finite difference numerical method consists of the following

steps:

Divide the solution region into a square 12 number of elements.

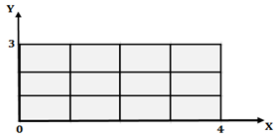

Figure 2: Finite difference arrangement for heat conduction problem.

- The implementation of the Gauss-Seidel method for solving the finite difference equations. There are 6 interior grid points that need to be determined.
- For a grid having equal horizontal and vertical step sizes, the interior grid points are given by finite difference equation.

$$X_{i,j} = \frac{1}{4}[x_{i+1,j} + x_{i-1,j} + x_{i,j+1} + x_{i,j-1}]$$

Iteration	T1	T2	T3	T4	T5	T6
1	0	0	0	0	0	0
2	13.18532731	4.666857682	3.222181333	15.79633183	5.115797377	2.084494678
3	18.29829738	8.023919022	4.574078425	18.35352369	7.115484347	2.922390693
4	19.77686068	9.231455862	5.085436639	19.22308626	7.844233203	3.23241746
5	20.29613553	9.671301343	5.272904701	19.53509218	8.109702747	3.345651862
6	20.48409838	9.831526457	5.34126958	19.64845028	8.20640715	3.386919183
7	20.55249418	9.889892729	5.366177978	19.68972533	8.241634311	3.401953072
8	20.57740452	9.911154201	5.375251818	19.70475971	8.254466745	3.407429641
9	20.58647848	9.91889926	5.378557225	19.71023631	8.259141302	3.409424632
10	20.58978389	9.921720605	5.379761309	19.7122313	8.260844134	3.410151361

Table 1: Solution by finite difference method

b. Finite Element Method

The spreadsheet implementation of the finite element numerical method consists of the following steps:

- Divide the solution region into a triangular 24 number of elements.

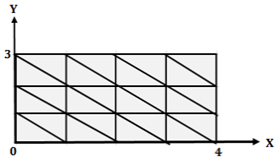

Figure 3: Finite element arrangement for heat conduction problem.

- Generate the input data and consists of three tables: global node x and y coordinates; global and local node correspondence for each element; and list of nodes with fixed temperature.
- For each element, the following quantities are computed and obtain the element coefficient matrix.

$$(x_3 - x_2, y_2 - y_3), \quad (x_1 - x_3, y_3 - y_1), \quad (x_2 - x_1, y_1 - y_2)$$

- The VLOOKUP function is invoked to retrieve global node coordinates and assembled for the global coefficient matrix C.
- The matrices C_f (free nodes) and C_{ff} (free and fixed nodes) are formed by extracting the appropriate rows and columns from the global coefficient matrix C.
- The final solution is obtained by using the matrix capabilities of Microsoft Excel.

Nodes	V_f
T1	19.1324
T2	8.7832
T3	6.0012
T4	20.2341
T5	7.2789
T6	4.5612

Table 1: Solution by finite element method

4. Discussion and Results.

As indicated in Table 3, the temperature at the free nodes computed by finite element and finite difference numerical grid method of this heat conduction problem compared fairly well.

Nodes	FEM	FDM
T1	19.1324	20.5968
T2	8.7832	9.9330
T3	6.0012	5.3914
T4	20.2341	19.7152
T5	7.2789	8.2651
T6	4.5612	3.4139

Table 3: Solution by finite element and finite difference method

The accuracy of the finite element numerical solution can be improved if a finer triangular grid is used.

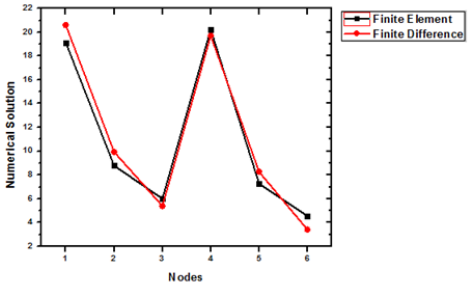

Figure 4: Compare the FD and FE numerical solution.

5. Conclusion

This paper presented the finite element and finite difference numerical solution using MS Excel for solving steady state heat conduction problem. The spreadsheet approach is ideal if the emphasis is on understanding of finite element and finite difference numerical grid technique. After comparing finite element numerical solution with finite difference numerical solution, the obtained. The Finite element numerical solution is fairly well.

References

1]. Kreyszig, E. Advanced engineering mathematics, 9th edition, John Wiley & Sons, 2006.

2]. Lau, M. A. and Kuruganty, S. P. Spreadsheet implementations for solving power-flow problems, eJournal of Spreadsheets in Education, 3(1): 27-45, 2008.

3]. O'Neil, P. Advanced engineering mathematics, 6th edition, CL-Engineering, 2006.

4]. Sadiku, M. Elements of electromagnetics, 4th edition, Oxford University Press, 2006.

5]. Yamani, A. and Kharab, A. Use of a spreadsheet program in electromagnetics, IEEE Transactions on Education, 44(3): 292-297, 2001.

6]. Lau, M. A. and Kuruganty, S. P. Spreadsheet Implementations for Solving Boundary-Value Problems in Electromagnetics, eJournal of Spreadsheets in Education, 4(1), 2010.